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Conditional Probability

P(A|B) can be said as:

- Probability of occurence of event A when B has already occurent
- Probability of occurence of event A when B is taken as sample space.

अब तक हमने probability का एक formulae पढ़ा था i.e. P(event A) = favourabe outcomes to A divided by total no. of outcomes. पर अब हम उसमें एक condition डालेंगे जो उस condn. को हमने ऊपर लिखा है। अब समझिएगा की उन दोनों में slight difference है

तो सबसे पहले समझते है इस definition को:-

⇒ P(A|B) = Probability of occurence of event A when event B has already occured.

इसके लिए example लिए :-

एक bag में 4 black balls & 5 red balls हैं तो जो पहला ball draw किए वह red है, क्या probability है कि दूसरा ball is black?

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तो इस question का solution कुछ ऐसा होगा:-

Bag

let,

A = 4 black balls है 1 भ.
B = 5 Red balls है

तो जब पहला event i.e. drawing a red ball हुआ तो कुल 9 balls मेरे पास थे जिसमें से सिर्फ 8 बचे

इसलिए, probability that second ball drawn is black.

$= \frac{4}{8}$ → Favourable / → Total.

क्योंकी एक ball निकाल चुके थे इस event से पहले इसलिए अब sample space में 8 balls होंगी।

⇒ अब हम समझते है दूसरा definition को :-

P(A|B) = Probability of occurence of event A when event B is taken as sample space.

(P.T.O.)

इसके लिए example लिए :-

हमने 2 dice को roll किया

तो हमने event ऐसे define किए :-

A = sum is 8

B = even number on first dice

अब देखो

A event के लिए possibilities :-

{(6,2), (5,3), (4,4)
(3,5), (2,6)}

B event के लिए possibilities :-

{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

अब देखो इस experiment के लिए जो original sample space बनेगा उसमें 36 elements होगा।

तो अब देखो :-

P(A|B) = probability of A when B is taken as sample space.

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तो अब sample space event B को लेंगे यानी :-

$S = \{ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6) \}$

18 outcomes

favourable outcome for event A

$= \{ (6,2),\ (5,3),\ (4,4),\ (3,5),\ (2,6) \}$

इसलिए

$$P(A \mid B) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{\cancel{3}}{\cancel{18}\ 6} = \frac{1}{6}$$

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अब हमने जब theoretically conditional probability के concept को समझ लिया है। तो आइए अब इसका formulae समझे :-

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \;;\; P(B) \neq 0$$

(or),

$$P(B|A) = \frac{P(A\cap B)}{P(A)} \;;\; P(A) \neq 0$$

Q. Let A and B be the events such that

$P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A\cap B) = \frac{4}{13}$.

find :-

(i.) $P(A|B)$
(ii.) $P(B|A)$
(iii.) $P(A\cup B)$
(iv.) $P(\bar{B}|\bar{A})$

Ans: दिया गया है।

A और B events है such that

$P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ & $P(A\cap B) = \frac{4}{13}$

(i.) $P(A|B)$

$$= \frac{P(A\cap B)}{P(B)} = \frac{4}{13} \times \frac{13}{9} = \frac{4}{9}$$

$\therefore P(A|B) = \frac{4}{9}$.

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(ii.) P(B|A)

$$= \frac{P(A \cap B)}{P(A)}$$

$$= \frac{4}{13} \times \frac{13}{7} = \frac{4}{7}$$

$\therefore P(B/A) = \frac{4}{7}$.

(iii.) P(A∪B)

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{7}{13} + \frac{9}{13} - \frac{4}{13}$$

$$= \frac{12}{13}$$

(iv.) $P(\bar{B}/\bar{A})$

$$= \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$$

by using de-morgan's law,
$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

$$= \frac{P(\overline{A \cup B})}{P(\bar{A})}$$

$$= \frac{1 - P(A \cup B)}{1 - P(A)}$$

$$= \frac{1 - \frac{12}{13}}{1 - \frac{7}{13}} = \frac{1}{6}.$$

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Q. Let A and B be the events such that $P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ and $P(A \cup B) = \frac{7}{11}$.

find :-

(i.) $P(A \cap B)$ (iii.) $P(B|A)$

(ii.) $P(A|B)$ (iv.) $P(\bar{A}/\bar{B})$

Ans: दिया हुआ है कि :-

A and B be the events such that

$P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ and

$P(A \cup B) = \frac{7}{11}$

(i.) $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A \cap B) = \frac{5}{11} + \frac{6}{11} - \frac{7}{11}$

$= \frac{4}{11}$

(ii.) $P(A|B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{4}{\cancel{11}} \times \frac{\cancel{11}}{6}$

$= \frac{\cancel{4}\,2}{\cancel{6}\,3}$

$= \frac{2}{3}$

(iii) $P(B|A)$

$$= \frac{P(A \cap B)}{P(A)} = \frac{4}{11} \times \frac{11}{5} = \frac{4}{5}$$

(iv) $P(\bar{A}|\bar{B})$

$$= \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$$

$$= \frac{P(\overline{A \cup B})}{P(\bar{B})}$$

$$= \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - \frac{7}{11}}{1 - \frac{6}{11}}$$

$$= \frac{\frac{11-7}{11}}{\frac{11-6}{11}}$$

$$= \frac{4}{5}$$

Q. Let A and B be the events such that

$P(A) = \frac{3}{10}$, $P(B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{5}$.

Find :-

(i.) $P(A \cap B)$ (ii.) $P(A \cup B)$ (iii.) $P(A|B)$

Ans: दिया गया है कि

A & B be the events, such that

$P(A) = \frac{3}{10}$, $P(B) = \frac{1}{2}$. & $P(B|A) = \frac{2}{5}$.

(i) $P(A \cap B)$

$\because P(B|A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow P(A \cap B) = P(B|A) \cdot P(A)$.

$\Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{3}{10} = \frac{3}{25}$

(ii) $P(A \cup B)$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{3}{10} + \frac{1}{2} - \frac{3}{25}$

$\Rightarrow P(A \cup B) = \frac{15 + 25 - 6}{50}$

$= \frac{34}{50}$

(iii.) $P(A|B)$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{3}{25} \times 2 = \frac{6}{25}.$$

Q. let A and B be the events such that :-

$2 \cdot P(A) = P(B) = \frac{5}{13}$ and $P(A|B) = \frac{2}{5}$.

find

(i.) $P(A \cap B)$ (ii.) $P(A \cup B)$

Ans: दिया गया है कि

A & B be the events

such that

$$2 \cdot P(A) = P(B) = \frac{5}{13} \quad \&$$

$$P(A|B) = \frac{2}{5}.$$

(i.) $P(A \cap B)$

$$\because P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$$

$$\Rightarrow P(A \cap B) = \frac{2}{5} \times \frac{5}{13}$$

$$= \frac{2}{13}$$

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(ii) P(AUB)

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$

$= \frac{5+10-4}{26} = \frac{11}{26}$

Q. A die is rolled. If the outcomes is an even number, what is the probability that it is a number greater than 2?

Ans: एक die में 6 faces होते हैं और इसका sample space = {1, 2, 3, 4, 5, 6}

Total no. of outcomes = 6

मान लिया कि P(A) probability है for getting an even number.

The sample space of A = {2, 4, 6}

$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$

मान लिया कि P(B) probability है for getting a number whose value is greater than 2.

The sample space of B = {3, 4, 5, 6}

$\therefore (A \cap B) = \{4, 6\}$

$\therefore P(A \cap B) = \frac{2}{6} = \frac{1}{3}.$

by conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

जहाँ, P(A|B) is the probability of occurrence of the event A given that B event has already occurred.

The probability of getting a number greater than 2 given that the outcome is even is given by :–

$P(B|A)$

$$= \frac{P(A \cap B)}{P(A)}$$

$$= \frac{1/3}{1/2} = \frac{2}{3}.$$

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Q. A coin is tossed twice. If the outcome is atmost one tail, what is the probability that both head and tail have appeared?

Ans: एक coin के पास 2 sides होते हैं और sample space $S = \{H, T\}$

The total no. of outcomes = 2

~~एक coin को दो बार उछाला~~

माना लिया, P(A) be the probability of getting at most 1 tail.

तो sample space of $A = \{(H,H), (H,T), (T,H)\}$

माना लिया, P(B) be the probability of getting a head.

तो sample space of $B = \{H\}$.

$\therefore P(B) = \frac{1}{2}$.

अब देखो probability of getting at most one tail and a head

i.e. $(A \cap B) = \{(H,H)\}$

$\therefore P(A \cap B) = \frac{1}{3}$

$P(A|B) \rightarrow$ probability of occurrence of the event A given that B has already occured.

The probability that both head and tail have appeared:

$P(A|B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/3}{1/2} = \frac{2}{3}$

Q. Three coins are tossed simultaneously. Find the probability that all coins show heads if at least one of the coin shows a head.

Ans: जब तीन coins को एक साथ toss किए तो total no. of outcomes $= 2^3 = 8$

और

sample space होगा।

$S = \{ (H,H,H), (H,H,T), (H,T,T), (H,T,H), (T,H,T), (T,T,H), (T,H,H), (T,T,T) \}$

माना किया $P(A)$ be the probability of getting 3 heads.

The sample space of A = {(H, H, H)}

$\therefore P(A) = \frac{1}{8}$

माना किया $P(B)$ be the probability of getting at least 1 head.

Probability of one head = 1 − probability of no heads

$= 1 - \frac{1}{8} = \frac{7}{8}$.

$\therefore P(B) = \frac{7}{8}$.

The probability that the throw is either all heads or at least one head

i.e.

$P(A \cup B) = \frac{7}{8}$

अब,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A \cap B) = \frac{1}{8} + \frac{7}{8} - \frac{7}{8}$

$= \frac{1}{8}$

$P(A|B)$ = The probability that all coins show heads if at least one of the coins showed a head.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/8}{7/8} = \frac{1}{7}$$

Q. Two unbaised dice are thrown. Find the probability that the sum of the numbers appearing is 8 or greater, if 4 appears on the first die.

Ans: दो die हर एक के पास 6 faces है उनको एक साथ उछाला। तो मेरे पास

total no. of outcomes $= 6^2 = 36$.

मान लिया

$P(A)$ be the probability of getting a sum greater than 8.

$P(B)$ be the probability of getting 4 on the first die.

B के लिए sample space

$= \{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}$

$\therefore P(B) = \frac{6}{36} = \frac{1}{6}$

माना लिया कि $P(A \cap B)$ है probability of getting 4 on the first die and the sum greater than or equal to 8.

तो $(A \cap B)$ के लिए sample space होगा :-

$\{(4,4), (4,5), (4,6)\}$

$\therefore P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

Now,

$P(A|B)$ = probability that sum of the no.s is greater than or equal to 8 given that 4 was thrown first.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{12}}{\frac{1}{6}}$$

$$= \frac{1}{2}.$$

(P.T.O.)

Q. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once?

Ans: एक dice जिसको दो बार फेंके उसके पास total outcomes होंगे $6^2 = 36$

माना लिया,

P(A) be the probability of getting the number 5 at least once.

P(B) be the probability of getting sum of 8.

The sample space of B

$= \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$\therefore P(B) = \frac{5}{36}$

माना लिया कि $P(A \cap B)$ be the probability of getting the no. 5 at least once and the sum equal to 8.

The sample space of $(A \cap B)$

$= \{(3,5), (5,3)\}$

$$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

using conditional probability

P(A|B) = The probability that the no. 5 have appeared at least once given that the sum is 8.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/18}{5/36} = \frac{36^{2}}{18 \times 5} = \frac{2}{5}$$

Q. Two dice were thrown and it is known that the numbers which come up were different. Find the probability that the sum of the two numbers was 5.

Ans: दो die जिसके पास 6 faces हैं हर एक dice पर उसको हम एक साथ उछालें तो total outcomes होंगे $6^2 = 36$

माना लिया

P(A) be the probability of getting a sum equal to 5.

P(B) be the probability of getting 2 different numbers.

Probability of getting 2 different numbers

= 1- probability of getting same no.s

$= 1 - \frac{\cancel{3}6}{\cancel{36} \cdot 36}$

$\Rightarrow 1 - \frac{1}{6} = \frac{5}{6}$.

$\therefore P(B) = \frac{5}{6}$

मान लिया कि $P(A \cap B)$ be the probability of getting a sum = 5 & two different numbers at the same time.

The sample space of $(A \cap B)$

$= \{(1,4), (2,3), (3,2), (4,1)\}$

$\therefore P(A \cap B) = \frac{4}{36} = \frac{1}{9}$

$P(A|B)$ = The probability that the sum = 5 given that two different no.s were thrown.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/9}{5/6} = \frac{2}{15}.$$

Q. A coin is tossed and then a die is thrown. Find the probability of obtaining a 6, given that a head came up.

Ans: एक coin को toss किए उसके बाद एक die को throw किए।

एक coin जिसमें 2 sides होते हैं उसमें total 2 outcomes होते हैं viz. {H, T}

एक die में 6 faces हैं इसलिए उसमें total 6 outcomes होंगे i.e. {1, 2, 3, 4, 5, 6}

मान लिया P(A) be the probability of getting the number 6

$$\therefore P(A) = \frac{1}{6}$$

मान लिया P(B) be the probability of getting a head.

The sample space of B = {H}

$$\therefore P(B) = \frac{1}{2}$$

मान लिया P(A∩B) be the probability of getting the number 6 and a head.

$$\therefore P(A \cap B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

(P.T.O.)

Using conditional probability

$P(A|B)$ = The probability that 6 come up given that head come up

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/12}{1/2} = \frac{1}{6}.$$

Q. A couple has 2 children. Find the probability that both are boys if it is known that

(i) One of the children is a boy, and
(ii) the elder child is a boy

Ans: एक couple के पास दो children हैं।

The sample space

$S = \{(B, B), (B, G), (G, B), (G, G)\}$

मान लो कि $P(A)$ be the probability of both being boys.

(P.T.O.)

(i) One of the children is a boy

माना कि P(B) be the probability of one of them being a boy.

The sample space of B

$$= \{(B,B), (B,G), (G,B)\}$$

$$\therefore P(B) = \frac{3}{4}.$$

मान लिया कि $P(A \cap B)$ be the probability of one of them being a boy and both being boys.

$$\therefore A \cap B = \{(B,B)\}$$

$$\therefore P(A \cap B) = \frac{1}{4}.$$

by using conditional probability :-

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{3/4} = \frac{1}{3}$$

$\therefore$ Probability that both are boys if it is known as that one of the children is a boy

$$= \frac{1}{3}.$$

(ii) the elder child is a boy

मान लिया कि P(B) be the probability of the elder being a boy.

The sample space of B

$= \{(B, B), (B, G)\}$

$\therefore P(B) = \frac{1}{2}$

मान लिया कि $P(A \cap B)$ be the probability of the elder being a boy and both being boys.

$\therefore A \cap B = \{(B, B)\}$

$\therefore P(A \cap B) = \frac{1}{4}$

$P(A|B)$ = probability of both being boy when the elder child is a boy.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/4}{1/2} = \frac{2}{4} = \frac{1}{2}$$

Q) In a class, 40% students study mathematics; 25% study biology and 15% study both maths and biology. One student is selected at random. Find the probability that :-

(i) he studies mathematics if it is known that he studies biology.
(ii) he studies biology if it is known that he studies mathematics.

Ans: माना कि P(A) be the probability of students studying mathematics.

$$\therefore P(A) = 40\% = \frac{40}{100} = 0.4$$

माना कि P(B) be the probability of students studying biology.

$$\therefore P(B) = 25\% = \frac{25}{100} = 0.25$$

माना कि $P(A \cap B)$ be the probability of students studying both mathematics and biology.

$$\therefore P(A \cap B) = 0.15$$

One student is selected at random.

$\because$ By conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where P(A|B) is the probability of occurrence of event A given that B has already occurred.

(i) he studies mathematics if it is known that he studies biology.

P(A|B) = probability that he studies mathematics given that he studies biology.

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.15}{0.25} = \frac{3}{5}$$

(ii) he studies biology if it is known that he studies mathematics.

P(A|B) = probability that he studies biology given that he studies mathematics.

$$= P(A|B)$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.15}{0.40}$$

$$= \frac{3}{8}$$

Q. The probability that a student selected at random from a class will pass in Hindi is $\frac{4}{5}$ and the probability that he passes in Hindi and English is $\frac{1}{2}$. What is the probability that he will pass in English if it is known that he has passed in Hindi ?

Ans: One student is selected at random.

माना P(A) be the probability of students passing in English.

माना P(B) be the probability of students passing in hindi.

$$\therefore P(B) = \frac{4}{5}$$

मान लिए कि $P(A \cap B)$ be the probability of students passing in both English & Hindi.

$$\therefore P(A \cap B) = \frac{1}{2}$$

by conditional probability

$P(A|B)$ = The probability that he will pass in English given that he passes in Hindi

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{1/2}{4/5} = \frac{5}{8}$$

Q. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a coat is 0.3 and the probability that he will buy a shirt given that he buys a coat is 0.4. Find the probability that he will buy both a shirt and a coat.

Ans: माना कि P(A) be the probability of a certain person buying a shirt.

$\therefore P(A) = 0.2$

माना P(B) be the probability of him buying a coat.

$\therefore P(B) = 0.3$

मान लिया कि $P(A \cap B)$ be the probability that he buys both a shirt and a coat.

by conditional probability

P(A|B) = the probability that he will buy a shirt given that he buys a coat.

$$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= 0.4$$

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$\Rightarrow P(A \cap B) = P(A|B) \cdot P(B)$
$= 0.4 \times 0.3$
$= 0.12$

Q. In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(i) Find the probability that he reads neither Hindi nor English news paper.

(ii) If he reads Hindi newspaper, what is the probability that he reads English newspaper?

(iii) If he reads English newspaper, what is the probability that he reads Hindi newspaper?

Ans: माना की P(A) be the probability of students reading hindi newspaper.

$\therefore P(A) = 60\% = \frac{60}{100} = 0.60$

माना की P(B) be the probability of them reading english newspaper.

$\therefore P(B) = 40\% = \frac{40}{100} = 0.40$

(P.T.O.)

माना की $P(A \cap B)$ be the probability of them reading both.

$\therefore P(A \cap B) = 0.20$.

मान लिए कि $P(A \cup B)$ be the probability of them reading either one of them.

$\therefore P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$
$= 0.60 + 0.40 - 0.20$
$= 1 - 0.20$
$= 0.80$

(ii) The probability that he reads neither Hindi nor English newspaper

$= 1 -$ probability that ~~either~~ of them reading either one of them

$= 1 - 0.8$
$= 0.2$
$= \frac{1}{5}$

(P.T.O.)

(ii) The probability that she reads the english one given that she reads the hindi one

by using conditional probability

~~P(A|B)~~ P(B|A)

$$= \frac{P(A\cap B)}{P(A)} = \frac{0.20}{0.60} = \frac{1}{3}.$$

(iii) The probability that she reads the Hindi one given that she reads the English one

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.20}{0.40} = \frac{1}{2}$$

Q. Two integers are selected at random from integers 1 through 11. If the sum is even, find the probability that both the numbers selected are odd.

Ans: Two integers are selected at random.

पहला choice के पास 11 options हैं 11 integers में से & second choice के पास 10 options हैं बचे 10 integers में से

माना लिया कि, P(A) be the probability of choosing both numbers odd.

P(B) be the probability of choosing the numbers to yield an even number.

Sample Space of B

$\{(1,3)\ (1,5)\ (1,7)\ (1,9)\ (1,11)\ (3,5)\ (3,7)$
$(3,9)\ (3,11)\ (5,7)\ (5,9)\ (5,11)\ (7,9)\ (7,11)$
$(9,11), (2,4), (2,6), (2,8), (2,10), (4,6),$
$(4,8), (4,10), (6,8), (6,10), (8,10)\}$

$$\therefore P(B) = \frac{25}{11 \times 10} = \frac{25}{110}$$

~~माना कि~~ P(A∩B) be the probability of getting both odd numbers given that sum of digits is even.

$\therefore (A \cap B) =$

$\{(1,3), (1,5), (1,7), (1,9), (1,11), (3,5)$
$, (3,7), (3,9), (3,11), (5,7), (5,9)$
$, (5,11), (7,9), (7,11), (9,11)\}$

Properties of Conditional Probability :-

Let A and B be the events of a sample space 'S' of an experiment, then

(i) $P(S|A) = P(A|A) = 1$

(ii) If A and B are any two events of a sample space S and C is an event of S, such that $P(C) \neq 0$, then

$$P\{(A \cup B)/C\} = P(A|C) + P(B|C) - P\{(A \cap B)|C\}$$

In particular, if A and B are disjoint events, then

$$P\left(\frac{A \cup B}{C}\right) = P\left(\frac{A}{C}\right) + P\left(\frac{B}{C}\right)$$

(iii) $P(A'|B) = 1 - P(A|B)$, where A' is complement of A.

Note: If A and B are mutually exclusive events, then $P(A|B) = 0$

Multiplication theorem on Probability and Independent Events

मान लिया की A and B दो events हैं जो एक random experiment से associated हैं, तब

$$P(A \cap B) = P(A) \cdot P(B|A), \text{ where } P(A) \neq 0$$
$$= P(B) \cdot P(A|B), \text{ where } P(B) \neq 0$$

यहाँ, $A \cap B$ denote करती है simultaneous occurrence of the events A and B i.e. $A \cap B$ जो denote करता है कि ऐसा event जिसमें दोनों A और B occur हुआ हो।

उपरोक्त result को हम "Multiplication rule of Probability".

✯ Multiplication Theorem for more than Two Events

Let E, F, G and H be four events of sample space S. Then,

(i) $P(E \cap F \cap G) = P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right)$

(ii) $P(E \cap F \cap G \cap H)$

$$= P(E) \cdot P\left(\frac{F}{E}\right) \cdot P\left(\frac{G}{E \cap F}\right) \cdot P\left(\frac{H}{E \cap F \cap H}\right)$$

Independent Events

दो events A & B को independent कहेंगे अगर occurrence or non-occurrence of एक event का दूसरे event को affect नहीं करे।

दो events E and F are said to be independent, if

$$P\left(\frac{F}{E}\right) = P(F), \text{ provided } P(E) \neq 0$$

and

$$P\left(\frac{E}{F}\right) = P(E), \text{ provided } P(F) \neq 0$$

दूसरे शब्दों में कहें तो :-

अगर E & F दो events हैं एक ही same random experiment के तो E & F को हम independent कहेंगे,

if $P(E \cap F) = P(E) \cdot P(F)$.

(P.T.O.)

Important Results :-

1. दो events E and F को हम independent कहेंगे अगर वो independent नहीं है
   i.e. $P(E \cap F) \neq P(E) \cdot P(F)$

2. दो experiments को independent तब बोलेंगे जब हर एक pair of events E and F, जहाँ E को पहला experiment का event है & F जो है वह दूसरा experiment का even है तो probability of the simultaneous occurrence of the events E and F तो होगा product P(E) और P(F) का जिसको हम seperately calculate किए दो experiments के basis पर

   i.e. $P(E \cap F) = P(E) \cdot P(F)$

3. Three events A, B and C को mutually independent कहेंगे, अगर

$$P(A \cap B) = P(A) \cdot P(B)$$
$$P(A \cap C) = P(A) \cdot P(C)$$
$$P(B \cap C) = P(B) \cdot P(C)$$
and $$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

अगर ऊपर दिए गए चार में कोई भी सही नहीं हो तीन दिए गए events के लिए तो दिए गए events "independent नहीं" होंगे i.e. dependent होंगे।

Difference between Independent events & Mutually exclusive events

(i.) "independent" term को हम define करते है probability of events के terms में लेकिन "mutually exclusive" को हम define करते है जब हम बात करते है कि जब events subset है sample space का।

(ii.) Mutually exclusive events के पास कोई भी outcome common नहीं होता है लेकिन independent events के पास हो सकता है common outcome हो जाए

दूसरे शब्दों में :-
दो independent events जिनके पास non-zero probability of occurrence है वह कभी भी independent नहीं होती।

Properties of Independent Events :-

अगर A और B independent events है, तो

(i.) A और B' भी independent events होंगी।

(ii.) A' और B भी independent events होंगी।

(iii.) A' और B' को भी independent events कहेंगे।

(P.T.O.)

Q. A bag contains 17 tickets, numbered from 1 to 17. A ticket is drawn, and then another ticket is drawn without replacing the first one. Find the probability that both the tickets may show even numbers.

Ans: एक बैग में 17 tickets हैं जिसमें 1 से 17 नंबर अंकित हैं और हर एक trial एक दूसरे से independent होते।

इसलिए sample space हुआ

$$S = \{1, 2, 3, \ldots\ldots\ldots, 17\}$$

हमको निकालना है the probability that both the tickets are drawn show even numbers.

तो ticket पर even no. आए इस event के लिए favourable outcome होंगे :-

$$\{2, 4, 6, 8, 10, 12, 14, 16\}$$

मान लिया, success : ticket drawn is even

अब, probability of success in the first trial

$$P_1(\text{success}) = \frac{8}{17}$$

probability of success in the second trial without replacement of the first draw is given by

$$P_2(\text{success}) = \frac{7}{16}$$

इसलिए,

probability that both the tickets show even numbers with each trial being independent is given by

$$P_1 \cap P_2 = P_1 \cdot P_2$$

$$= \frac{8}{17} \times \frac{7}{16_2} = \frac{7}{34}$$

Q. Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the probability that both the marbles are black if the first marble is not replaced before the second draw.

Ans: दिया हुआ है कि :-
एक box में 3 black और 4 white marbles है, हर एक trial एक दूसरे से independent है।

इसलिए sample space होगा

$$S = \{B_1, B_2, B_3, W_1, W_2, W_3, W_4\}$$

(P.T.O.)

तो हमको निकालना है कि probability that both the marbles drawn are black

माना कि, success : marble drawn is black

अत, the probability of success in the first trial is

$$P_1(\text{success}) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{\{B_1, B_2, B_3\}}{\{B_1, B_2, B_3, W_1, W_2, W_3, W_4\}} = \frac{3}{7}$$

Probability of success in the second trial without replacement of the first draw is given by

$$P_2(\text{success}) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

$$= \frac{2}{6}$$ ($\because$ without replacement we are drawing again)

Hence,
the probability that both the marbles are drawn are black, with each trial being independent is given by

$$P(\text{success}_1 \cap \text{success}_2) = P_1(\text{success}) \times P_2(\text{success})$$

$$= \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$$

Q. A card is drawn from a well-shuffled deck of 52 cards and without replacing this card, a second card is drawn. Find the probability that the first card is a club and the second card is a spade.

Ans: दिया हुआ है कि a well shuffled deck of 52 cards जिसमें हर draw एक दूसरे से independent होता है।

हमें निकालना है the probability that the first card is drawn is a club and the second card is a spade.

मान लिया कि

E: event that first card drawn is a club

H: event that second card drawn is a spade

तो

$$P(E) = \frac{13}{52}$$ {∵ there is total of 13 cards in club suite

$$P(H) = \frac{13}{51}$$ {∵ there is total of 13 cards in spade suite & one card is already drawn

इसलिए,

the probability that the first card drawn is a club and the second card is a spade, with each trial being independent is given by

$$P(E \cap H) = P(E) \cdot P(H)$$

$$= \frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$$

Q. There is a box containing 30 bulbs, of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs are chosen are defective?

Ans: दिया हुआ है कि एक box है जिसमें 30 bulbs हैं जिसमें 5 defective है, each trial एक दूसरे से independent है

तो हमको यह निकालना है कि the probability that both the bulbs are chosen are defective.

मान लिया कि

L: event that first ~~ball~~ bulb drawn is defective.

M: event that second bulb drawn is defective.

(P.T.O.)

तो

$$P(L) = \frac{5}{30}$$ ∵ there are 5 defective balls and in total there are 30 bulbs.

$$P(M) = \frac{4}{29}$$ ∵ there are remaining 4 defective bulbs and in total there are 29 bulbs

इसलिए,

the probability that both the bulbs are chosen are defective, with each trial being independent is given by

$$P(L \cap M) = P(L) \cdot P(M)$$

$$= \frac{5}{30} \times \frac{4}{29}$$

$$= \frac{2}{87}$$

Q. A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that the first ball is white and the second is black?

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Ans: दिया गया है एक bag में 10 white ball और 15 black balls है, each trial is independent of the other trial.

तो हमको निकालना है कि the probability that the first ball is drawn is white and the second ball drawn is black.

मान लिया,

E: event that first ball drawn is white.
M: event that second ball drawn is black.

$P(E) = \frac{10}{25}$ {∵ there are total 25 balls out of which there are only 10 white balls.

$P(M) = \frac{15}{24}$ {∵ there are total remaining 24 balls after first draw there are only 15 black balls.

Hence,

the probability that the first ball drawn is white & second ball drawn is black, with each trial being independent is given by.

$$P_1 \times P_2 = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}.$$

Q. An urn contains 5 white and 8 black balls. Two successive drawings of 3 balls at a time are made such that the balls drawn in the first draw are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and the second draw gives 3 black balls.

Ans: दिया गया है कि :-
एक कलश में 5 white balls और 8 black balls हैं; each trial is independent of the other trial.

तो हमको निकालना है, the probability that the first draws gives 3 white and the second draw gives 3 black balls.

तो माना कि :-

E : event that the first draw gives 3 white balls

F : event that the second draw gives 3 black balls

$$P(E) = \frac{\text{selecting 3 balls out of 5 white balls}}{\text{selecting 3 balls out of total 13 balls}}$$

$$= \frac{{}^5C_3}{{}^{13}C_3} = \frac{\frac{5!}{2! \cdot 3!}}{\frac{13!}{10! \cdot 3!}}$$

$$= \frac{5}{143}$$

$$P(F) = \frac{\text{selecting 3 balls out of 8 black balls}}{\text{selecting 3 balls out of 10 remaining total balls}}$$

$$= \frac{^{8}C_3}{^{10}C_3} = \frac{\frac{8!}{5! \cdot 3!}}{\frac{10!}{7! \cdot 3!}} = \frac{8! \times 7! \times 3!}{10! \times 5! \times 3!}$$

$$= \frac{8! \times 7 \times 6 \times 5!}{10 \times 9 \times 8! \times 5!} = \frac{42}{90} = \frac{21}{45} = \frac{7}{15}$$

$$= \frac{7}{15}$$

इसलिए :

the probability that the first draw gives 3 white and the second draw gives 3 black balls, with each trial being independent is given by

$$P(E \cap F) = P(E) \cdot P(F)$$

$$= \frac{5}{143} \times \frac{7}{15_3}$$

$$= \frac{7}{429}$$

Q. let E_1 and E_2 be the events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$.

find :-

(i) $P(E_1 \cup E_2)$, when E_1 and E_2 are mutually exclusive.

(ii) $P(E_1 \cap E_2)$, when E_1 and E_2 are independent.

Ans:

(i) $\because P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

since E_1 and E_2 are mutually exclusive events so, $E_1 \cap E_2 = \phi$

$$= \frac{1}{3} + \frac{3}{5} = \frac{5+9}{15} = \frac{14}{15}$$

$\therefore P(E_1 \cup E_2) = \frac{14}{15}$ when E_1 & E_2 are mutually exclusive.

(ii) $\because$ we know that E_1 and E_2 are independent.

so,

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$$

$$= \frac{1}{3} \times \frac{3}{5}$$

$$= \frac{1}{5}$$

Therefore, $P(E_1 \cap E_2) = \frac{1}{5}$ when E_1 and E_2 are independent.

Q. If E_1 and E_2 are the two events such that $P(E_1) = \frac{1}{4}$, $P(E_2) = \frac{1}{3}$ and $P(E_1 \cup E_2) = \frac{1}{2}$, show that E_1 and E_2 are independent events.

Ans: हमको बहुत अच्छे से पता है कि अगर दो event मान लिजिए A&B independent events हैं तो

$$P(A \cap B) = P(A) \cdot P(B) \text{ होगा}$$

पहले निकालो

$$\left[\because P(A \cap B) = P(A) + P(B) - P(A \cup B) \right]$$

$$P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$$

$$= \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$$

$$= \frac{3 + 4 - 6}{12} = \frac{1}{12}$$

2	4, 3, 2
2	2, 3, 1
3	1, 3, 1
	1, 1, 1

and,

$$P(E_1) \cdot P(E_2)$$

$$= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

(P.T.O.)

Now,

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{1}{12}.$$

Hence therefore it is proved that E_1 & E_2 are independent events.

Q. If E_1 and E_2 are independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$, find

(i) $P(E_1 \cap E_2)$
(ii) $P(E_1 \cup E_2)$
(iii) $P(\overline{E_1} \cap \overline{E_2})$
(iv) $P(\overline{E_1} \cap E_2)$

Ans:

(i) क्योंकि E_1 & E_2 independent events है।
इसलिए,

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

$$= 0.3 \times 0.4$$
$$= 0.12$$

(ii) $\because P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow 0.3 + 0.4 - 0.12$
$\Rightarrow 0.7 - 0.12$
$\Rightarrow 0.58$

(iii.) $P(\overline{E_1} \cap \overline{E_2})$

$\{\because A' \cap B' = (A \cup B)'$; using de morgan's law$\}$

$\Rightarrow P(\overline{E_1} \cap \overline{E_2})$

$\Rightarrow P(\overline{E_1 \cup E_2})$

$\Rightarrow 1 - P(E_1 \cup E_2)$

$\Rightarrow 1 - 0.58 = 0.42$

(iv.) $P(\overline{E_1} \cap E_2)$

$\Rightarrow P(\overline{E_1} \cap E_2)$

$\Rightarrow P(E_2) - P(E_1 \cap E_2)$

$\Rightarrow 0.4 - 0.12$

$\Rightarrow 0.28$

Q. Let A and B be the events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\text{not } A \text{ or not } B) = \frac{1}{4}$.

State wether A and B are

(i.) Mutually exclusive

(ii.) independent

(P.T.O.)

Ans: दिया गया है :—

$P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$

$P(\bar{A} \cup \bar{B}) = \frac{1}{4}$.

(i.) let consider that A & B be mutualy exclusive events

then $A \cap B = \phi$

i.e. $P(A \cup B) = P(A) + P(B) - \phi$

$P(A \cup B) = \frac{1}{2} + \frac{7}{12} -$ —— (i)

but

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = \frac{1}{4}$

$= \phi$

if we want

$P(A \cap B) = 1 - P(\overline{A \cap B})$

$= 1 - \frac{1}{4} = \frac{3}{4}$.

from here

$P(A \cup B) = \frac{1}{2} + \frac{7}{12} - \frac{3}{4}$ —— (ii)

Since, two distinctie values of P(A ∪ B) calculated in (i) & (ii) respectively contradicts.

So, ~~P(A)~~ A & B are not mutually exclusive events.

(ii.) if A & B would have to be independent events

then,

$$P(A \cap B) = P(A) \cdot P(B)$$

Now

$$P(A \cap B) = \frac{3}{4}$$

&

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}$$

$$\because \quad P(A \cap B) \neq P(A) \cdot P(B).$$

So,

A and B are not independent events.

(P.T.O.)

Q. Kamal and Vimal appeared for an interview for two vacancies. The probability of Kamal's selection is 1/3, and that of Vimal's selection is 1/5. Find the probability that only one of them will be selected.

Ans: मान लिया :-

A : event that Kamal is selected.
B : event that Vimal is selected.

therefore,

$$P(A) = \frac{1}{3} \; ; \; P(B) = \frac{1}{5}.$$

हमें निकालना है probability कि सिर्फ एक आदमी select होगा।

The probability that only one of them will be selected

Now,

P(only one of them is selected)

= P(A and not B) or P(B and not A)

$$= P(A \cap \bar{B}) + P(B \cap \bar{A})$$

(P.T.O.)

∵ we know very well that the properties of independent events are :-
if A & B are independent event, then :-
(A∩B') & (A'∩B) are also independent events.

$$= P(A) \times P(\overline{B}) + P(B) \times P(\overline{A})$$

$$= P(A) \times [1 - P(B)] + P(B) \times [1 - P(A)]$$

$$= \frac{1}{3} \times \left(1 - \frac{1}{5}\right) + \frac{1}{5} \times \left(1 - \frac{1}{3}\right)$$

$$= \frac{1}{3} \times \frac{4}{5} + \frac{1}{5} \times \frac{2}{3}$$

$$= \frac{4}{15} + \frac{2}{15} = \frac{6}{15} = \frac{2}{5}$$

Therefore,

The probability that only one of them will be selected is $\frac{2}{5}$.

Q. Arun and Ved appeared for an interview for two vacancies. The probability of Arun's selection is 1/4, and that of Ved's rejection is 2/3. Find the probability that at least one of them will be selected.

Ans: माना किया :-

E : event that arun get selected
Z : event that ved get selected

therefore,

$$P(E) = \frac{1}{4}, \quad P(Z) = \frac{2}{3}$$

$\because$ E & Z are independent events so E' & Z ; E & Z' are also independent events.

$$P(E') = 1 - P(E) = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

$$P(Z') = 1 - P(Z) = 1 - \frac{2}{3} = \frac{1}{3}$$

$\therefore$ Probability that at least one of them will be selected

$$= P(E \cap Z') + P(E' \cap Z) + P(E \cap Z)$$

= probability that arun selected & ved not selected (or)
probability that ved selected & arun not selected (or)

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probability that both arun & ved get selected.

$= P(E \cap Z') + P(E' \cap Z) + P(E \cap Z)$

$= \{P(E) - P(E \cap Z)\} + \{P(Z) - P(E \cap Z)\}$
$+ P(E \cap Z)$

$= \left\{\frac{1}{4} - P(E) \cdot P(Z)\right\} + \left\{P(Z) - P(E) \cdot P(Z)\right\}$
$+ P(E) \cdot P(Z)$

$= \left\{\frac{1}{4} - \frac{1}{4} \times \frac{2}{3}\right\} + \left\{\frac{1}{3} - \frac{1}{4} \times \frac{1}{3}\right\} + \left\{\frac{1}{4} \times \frac{2}{3}\right\}$

$= \left\{\frac{1}{4} - \frac{2}{12}\right\} + \left\{\frac{1}{3} - \frac{1}{12}\right\} + \frac{1}{6}$

$= \left\{\frac{1}{4} - \frac{1}{6}\right\} + \left\{\frac{1}{3} - \frac{1}{12}\right\} + \frac{1}{6}$

$= \left\{\frac{3-2}{12}\right\} + \left\{\frac{4-1}{12}\right\} + \frac{1}{6}$

$= \frac{1}{12} + \frac{3}{12} + \frac{1}{6} = \frac{4}{12} = \frac{1}{3} + \frac{1}{6}$

$= \frac{6+3}{18} = \frac{9}{18} = \frac{1}{2}$

Q. A and B appear for an interview for two vacancies in the same post. The probability of A's selection is 1/6 and that of B's selection is 1/4. Find the probability that :-

(i.) both of them are selected
(ii.) only one of them is selected
(iii.) none is selected
(iv.) at least one of them is selected.

Ans: माना लिया कि :-

E: event that ~~only~~ A get selected.
J: event that B get selected.

दिया गया है कि :-

$P(A) = \frac{1}{6}$; $P(B) = \frac{1}{4}$.

$P(\bar{A}) = 1 - P(A)$
$= 1 - \frac{1}{6}$
$= \frac{5}{6}$
$P(\bar{A}) = \frac{5}{6}$

$P(\bar{B}) = 1 - P(B)$
$= 1 - \frac{1}{4}$
$= \frac{4-1}{4}$
$= \frac{3}{4}$

(P.T.O.)

(i) both of them are selected

$\because$ A & B are independent events.

So,

$P(A \cap B)$ = probability of getting both of them selected.

$$= P(A) \cdot P(B)$$

$$= \frac{1}{6} \times \frac{1}{1} = \frac{1}{24}.$$

(ii) Only one of them is selected.

$$P(A' \cap B) + P(A \cap B')$$

$$= \{P(B) - P(A \cap B)\} + \{P(A) - P(A \cap B)\}$$

$$= \left(\frac{1}{4} - \frac{1}{24}\right) + \left(\frac{1}{6} - \frac{1}{24}\right).$$

$$= \left(\frac{6-1}{24}\right) + \left(\frac{4-1}{24}\right)$$

$$= \frac{5}{24} + \frac{3}{24} = \frac{8}{24} = \frac{1}{3}.$$

Therefore, the probability that only one of them is selected

$$= \frac{1}{3}$$

(iii) none is selected

Probability of none get selected

$= P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \cdot P(\bar{B})$

$= \frac{5}{6} \times \frac{3}{4}$

$= \frac{15}{24} = \frac{3}{8}$

[∵ A & B are independent events, so, A' & B' are also independent events.]

Therefore, the probability that atleast one of them is selected is $\frac{3}{8}$.

(iv) at least one of them is selected.

P(at least one of them is selected)

= P(selecting only A) + P(selecting only B) + P(selecting both A & B).

= P(A and not B) + P(B and not A) + P(A and B)

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$= P(A)\cdot P(\overline{B}) + P(B)\cdot P(\overline{A}) + P(A)\cdot P(B)$

$= \left(\frac{1}{6}\times\frac{3}{4}\right) + \left(\frac{1}{4}\times\frac{5}{6}\right) + \frac{1}{6}\times\frac{1}{4}.$

$= \frac{1}{8} + \frac{5}{24} + \frac{1}{24}$

$= \frac{3+5+1}{24} = \frac{9}{24} = \frac{3}{8}$

Q. Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is 3/5, find the probability that :—

(i) at least one of A and B will solve the problem

(ii) none of the two will solve the problem.

Ans:

Let,

E: event that A can solve the problem.
M: event that B can solve the problem.

$P(E) = \frac{2}{3}$

$P(\overline{E}) = 1 - \frac{2}{3} = \frac{1}{3}$

$P(M) = \frac{3}{5}$

$P(\overline{M}) = 1 - \frac{3}{5} = \frac{2}{5}$

Clearly, A & B are independent events
so, A' & B' are also independent events.

(i) at least one of A and B will solve the problem

अतः,

P(at least one of A and B will solve the problem)

= 1 − P(both are unable to solve the problem)

$= 1 - P(A' \cap B')$

$= 1 - P(\overline{A}) \times P(\overline{B})$

$= 1 - P(\overline{E}) \times P(\overline{M})$

$= 1 - \frac{1}{3} \times \frac{2}{5}$

$= 1 - \frac{2}{15} = \frac{13}{15}$

Therefore, atleast one of A and B will solve the problem is $\frac{13}{15}$.

(ii) none of the two will solve the problem:

अतः,

P(none of the two will solve the problem)

$= P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \cdot P(\bar{B})$

$= P(\bar{E}) \cdot P(\bar{M})$

$= \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$.

Q. A problem is given to three students whose chances of solving it are 1/4, 1/5 & 1/6, respectively. Find the probability that the problem is solved.

Ans: दिया हुआ है कि :–

let A: be the event that problem solved by first student.

B: be the event that problem solved by second student.

C: be the event that problem solved by third student.

(P.T.O.)

Now,

$$P(A) = \frac{1}{4},\ P(B) = \frac{1}{5}\ \&\ P(C) = \frac{1}{6}$$

$$P(\overline{A}) = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

$$P(\overline{B}) = 1 - \frac{1}{5} = \frac{5-1}{5} = \frac{4}{5}$$

$$P(\overline{C}) = 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}.$$

∴ The probability that the problem is solve[d]

P(the problem is solved)

= 1 − P(the problem is not solved)

$$= 1 - P(\overline{A} \cap \overline{B} \cap \overline{C})$$

$$= 1 - P(\overline{A}) \times P(\overline{B}) \times P(\overline{C})$$

$$= 1 - \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$$

$$= 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}.$$

∴ The probability that the problem is solved

$$= \frac{1}{2}.$$

Q. The probabilities of A, B, C solving a problem are 1/3, 1/4 and 1/6, respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

Ans: माना कि ~~A, B and C be three students~~

E : event that A solves the problem.
F : event that B solves the problem.
Z : event that C solves the problem.

$P(E) = \frac{1}{3}$, $P(F) = \frac{1}{4}$, $P(Z) = \frac{1}{6}$

$P(\bar{E}) = 1 - P(E)$	$P(\bar{F}) = 1 - P(F)$	$P(\bar{Z}) = 1 - P(Z)$
$= 1 - \frac{1}{3}$	$= 1 - \frac{1}{4}$	$= 1 - \frac{1}{6}$
$= \frac{3-1}{3}$	$= \frac{4-1}{4}$	$= \frac{6-1}{6}$
$= \frac{2}{3}$	$= \frac{3}{4}$	$= \frac{5}{6}$.

Now,

P(exactly one of them will solve it)

= P(A and not B and not C) +
P(B and not A and not C) +
P(C and not A and not B).

(P.T.O.)

$= P(E \cap \bar{F} \cap \bar{Z}) + P(\bar{E} \cap F \cap \bar{Z}) + P(\bar{E} \cap \bar{F} \cap Z)$

$\left\{ \because \text{ E, F, Z are independent events then E', F', Z' are also independent events} \right\}$

$= P(E) \times P(\bar{F}) \times P(\bar{Z}) + P(\bar{E}) \times P(F) \times P(\bar{Z})$
$+ P(\bar{E}) \times P(\bar{F}) \times P(Z)$

$= \left(\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}\right) + \left(\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6}\right) +$
$\left(\frac{2}{3} \times \frac{3}{4} \times \frac{1}{6}\right)$

2	24, 12, 12
2	12, 6, 6
3	6, 3, 3
2	2, 1, 1
	1, 1, 1

$= \frac{5}{24} + \frac{5}{12} + \frac{1}{12}$

$= \frac{5 + 10 + 2}{24} = \frac{17}{24}$

$= \left(\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}\right) + \left(\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6}\right) +$
$\left(\frac{1}{6} \times \frac{3}{4} \times \frac{2}{3}\right)$

$$= \frac{15}{72} + \frac{10}{72} + \frac{6}{72} = \frac{31}{72}.$$

Therefore,
The probability that exactly one of them will solve the problem is $\frac{31}{72}$.

Q. A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. Calculate the probability that:-

(i.) A, B and C all hit the target.
(ii.) B and C hit and A does not hit the target.

Ans:
(i.) Let,
K: event that A hit a target once.
R: event that B hit a target once.
M: event that C hit a target once.

$$P(K) = \frac{\text{no. of times A hit the shot}}{\text{total no. of shots}} = \frac{4}{5}$$

$$P(R) = \frac{\text{no. of times B hit the shot}}{\text{total no. of shots}} = \frac{3}{4}$$

$$P(M) = \frac{\text{no. of times C hit the shot}}{\text{total no. of shots}} = \frac{2}{3}$$

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Now,

$P(\overline{K}) = 1 - P(K)$

$= 1 - \frac{4}{5} = \frac{5-4}{5} = \frac{1}{5}$

$P(\overline{R}) = 1 - P(R)$

$= 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$

$P(\overline{M}) = 1 - P(M)$

$= 1 - \frac{2}{3} = \frac{3-2}{3} = \frac{1}{3}$

Clearly, ~~A, B & C~~ K, R & M are independent events so also K', R & M are independent events.

∴ Probability that A, B & C all hit the target

$= P(K \cap R \cap M)$

$= P(K) \cdot P(R) \cdot P(M)$

$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$

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Hence, The probability that A, B and C all hit the target is $\frac{2}{5}$.

(ii.)

Probability that B and C hit and A does not hit the target.

P(B and C hit and not A)

$= P(\bar{K} \cap R \cap M)$

$= P(\bar{K}) \times P(R) \times P(M)$

~~$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$~~

$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{1}{10}$

Hence, the probability that B and C hit and A does not hit the target is $\frac{1}{10}$.

Q. Neelam has offered physics, chemistry & mathematics in class XII. She estimates that her probabilities of receiving a grade A in these courses are 0.2, 0.3 and 0.9 respectively. Find the probabilities that Neelam recieves

(i.) all A grades
(ii.) no A grade
(iii.) exactly 2 A grades.

Ans: मान लिया कि:-

E : event that neelam getting A grade in physics.
F : event that neelam getting A grade in chemistry.
G : event that neelam getting A grade in mathematics.

$P(E) = 0.2$, $P(F) = 0.3$, $P(G) = 0.9$

$P(\overline{E}) = 1 - P(E)$
$= 1 - 0.2$
$= 0.8$

$P(\overline{F}) = 1 - P(F)$
$= 1 - 0.3$
$= 0.7$

$P(\overline{G}) = 1 - P(G)$
$= 1 - 0.9 = 0.1$.

(i) The probability that neelam gets all A grades

Here,
P(getting all A grades)
$= P(E \cap F \cap G)$
$= P(E) \cdot P(F) \cdot P(G)$
$= 0.2 \times 0.3 \times 0.9$

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$= 0.054$

$\therefore$ The probability that neelam gets all A grades is 0.054.

(ii) The probability that neelam gets no A grades

P(getting no A grade)

$= P(\overline{E} \cap \overline{F} \cap \overline{G})$

$= P(\overline{E}) \times P(\overline{F}) \times P(\overline{G})$

$= 0.8 \times 0.7 \times 0.1$

$= 0.056$

$\therefore$ The probability that neelam gets no A grades is 0.056.

(iii) The probability that neelam gets exactly 2 A grades

P(getting exactly 2 A grades)

$= P(\text{A and B and not C}) + P(\text{B and C and not A}) + P(\text{C and A and not B})$

$= P(E \cap F \cap \overline{G}) + P(F \cap G \cap \overline{E}) + P(G \cap E \cap \overline{F})$

$= [0.2 \times 0.3 \times 0.1] + [0.8 \times 0.9 \times 0.3]$
$+ [0.9 \times 0.2 \times 0.7]$

$= [0.006 + 0.216 + 0.126]$
$= 0.348$

∴ The probability that neelam gets exactly 2A grades is

0.348.

Q. An article manufactured by a company consists of two parts X and Y. In the process of manufacture of part X, 8 out of 100 parts may be defective. Similarly, 5 out of 100 parts of Y may be defective. Calculate the probability that the assembled product will not be defective.

Ans: माना लिया

R: is the event that a defective part manufactured in X.
Q: is the event that a defective part manufactured in Y.

Now,

$P(R) = \frac{8}{100}$

$P(Q) = \frac{5}{100}$

$P(\overline{R}) = 1 - P(\overline{R})$

$= 1 - \frac{8}{100}$

$= \frac{100-8}{100}$

$= \frac{92}{100}$

$P(\overline{Q}) = 1 - P(\overline{Q})$

$= 1 - \frac{5}{100}$

$= \frac{100-5}{100}$

$= \frac{95}{100}$

∴ Probability that the assembled product will not be defective

$= 1 - P(\text{product assembled to be defective})$

$= 1 - [P(\text{X and not Y}) + P(\text{Y and not X}) + P(\text{both X and Y})]$

$= 1 - [P(R \cap \overline{Q}) + P(Q \cap \overline{R}) + P(R \cap Q)]$

$= 1 - [P(R) \cdot P(\overline{Q}) + P(Q) \cdot P(\overline{R}) + P(R) \cdot P(Q)]$

$= 1 - \left[\frac{8}{100} \times \frac{95}{100} + \frac{5}{100} \times \frac{92}{100} + \frac{8}{100} \times \frac{5}{100}\right]$

$= 1 - \left[\frac{760}{10000} + \frac{460}{10000} + \frac{40}{10000}\right] = \frac{437}{500}$

Therefore,
The probability that the assembled product will not be defective is

$$\frac{437}{500}$$

Q. A town has two fire-extinguishing engines, functioning independently. The probability of availability of each engine when needed is 0.95. What is the probability that

(i) neither of them is available when needed?
(ii) an engine is available when needed?

Ans: मान लिया कि,

E : event that first F.E.E is available
G : event that second F.E.E is available
P(E) = 0.95 & P(G) = 0.95

(i) The probability that neither of them is available when needed.

P(not E and not G)

$= P(\bar{E} \cap \bar{G})$

$= P(\bar{E}) \times P(\bar{G})$

$= [1 - P(E)] \times [1 - P(G)]$

{∵ E & G are independent events then E' & G' are independent events}

$= [1 - 0.95] \times [1 - 0.95]$

$= 0.05 \times 0.05$

$= 0.0025$

∴ The probability that neither of them is available when needed

$= 0.0025$.

(ii.) The probability that an engine is available when needed:

Hence,

P(E and not G) Or P (not E and G)

$= P(E \cap \overline{G}) + P(\overline{E} \cap G)$

$= P(E) \cdot P(\overline{G}) + P(\overline{E}) \cdot P(G)$

$= 0.95 \times 0.05 + 0.05 \times 0.95$

$= 0.0475 + 0.0475$

$= 0.0950$

∴ The probability that an engine is available when needed is

$= 0.0950$.

Q. A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third components are 0·14, 0·10 and 0·05, respectively. What is the probability that the machine will fail?

Ans: मान लिया कि :-

E: event that the first component fails.
B: event that the second component fails.
Z: event that the third component fails.

$P(E) = 0{\cdot}14$; $P(B) = 0{\cdot}10$; $P(Z) = 0{\cdot}05$

$P(\bar{E}) = 1 - 0{\cdot}14 = 0{\cdot}86$

$P(\bar{B}) = 1 - 0{\cdot}10 = 0{\cdot}90$

$P(\bar{Z}) = 1 - P(Z) = 1 - 0{\cdot}05 = 0{\cdot}95$

Hence,

The probability that the machine will fail

~~$= P(\bar{E} \cap \bar{B} \cap \bar{Z})$~~

= 1 − P(the machine will function)

$= 1 - P(\text{all three components are working})$

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 - [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})]$

$= 1 - [P(\bar{E}) \times P(\bar{B}) \times P(\bar{Z})]$

$= 1 - [0.86 \times 0.90 \times 0.95]$

$= 1 - 0.7353$

$= 0.2647$

Therefore,
The probability that the machine will fail is 0.2647.

anti-

Q. An air-craft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

(P.T.O.)

Ans:

B : event of hitting the plane at the first shot.
Q : event of hitting the plane at the second shot.
R : event of hitting the plane at the third shot.
S : event of hitting the plane at the fourth shot.

$P(B) = 0.4$, $P(Q) = 0.3$, $P(R) = 0.2$ & $P(S) = 0.1$

$P(\bar{B}) = 1 - P(B)$
$= 1 - 0.4$
$= 0.6$

$P(\bar{Q}) = 1 - P(Q)$
$= 1 - 0.3$
$= 0.7$

$P(\bar{R}) = 1 - P(R)$
$= 1 - 0.2$
$= 0.8$

$P(\bar{S}) = 1 - 0.1$
$= 0.9$

The probability that atleast one shot hits the plane.

$\Rightarrow$ P(atleast one shot hits the plane)
= 1 − P(none of the shot hit the plane)

$= 1 - P(\bar{B} \cap \bar{Q} \cap \bar{R} \cap \bar{S})$

$= 1 - P(\bar{B}) \cdot P(\bar{Q}) \cdot P(\bar{R}) \cdot P(\bar{S})$

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$= 1 - 0.6 \times 0.7 \times 0.8 \times 0.9$

$= 1 - 0.3024$

$= 0.6976$

Therefore,

The probability that atleast one shot hits the plane is

$= 0.6976$